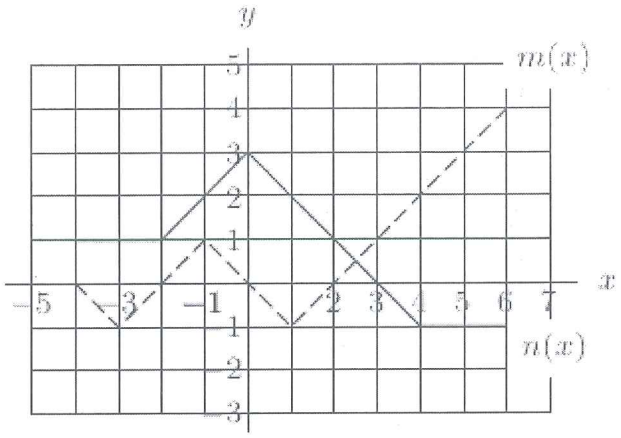
Chapter 10 Review Name: _____

All answers must be justified with work. No work, no credit!

1. The functions $m(x)$ and $n(x)$ are defined by the graph below. The dashed graph is $m(x)$ and the solid graph is $n(x)$.



Evaluate $m(n(5))$. $m(-1) = \boxed{1}$

2. If $H(x) = f(g(x)) = e^{6x+6}$, which of the following could be true? Mark all that apply.

A) $f(x) = e^x$, $g(x) = 6x+6$ e^{6x+6}

B) $f(x) = 6x+6$, $g(x) = e^x$ $6e^x + 6$

C) $f(x) = x^2$, $g(x) = e^{3x+3}$ $(e^{3x+3})^2 = e^{6x+6}$ {Power to a Power – multiply exponents}

D) $f(x) = e^{3x+3}$, $g(x) = x^2$ e^{3x^2+3}

E) $f(x) = x+6$, $g(x) = e^{6x}$ $e^{6x}+6$

F) $f(x) = e^{6x}$, $g(x) = x+6$ $e^{6(x+6)} = e^{6x+36}$

3. If $f(x) = x^2 + 6x$ and $g(x) = 8 - x$, Find $f(g(x))$.

$f(g(x)) = (8-x)^2 + 6(8-x)$

$= 64 - 16x + x^2 + 48 - 6x$

$\boxed{f(g(x)) = x^2 - 22x + 112}$

4. If $p(q(x)) = \dfrac{4}{1+x}$ and $q(x) = 6 + x$, what is $p(x)$?

$p(x) = \dfrac{4}{-5+x}$ or $p(x) = \dfrac{4}{x-5}$

$p(q(x)) = \dfrac{4}{6+x-5} = \dfrac{4}{1+x}$ ✓

5. Let $f(x) = \cos(7x)$ and $g(x) = \sqrt{1-x^2}$. What is $g(f(x))$?

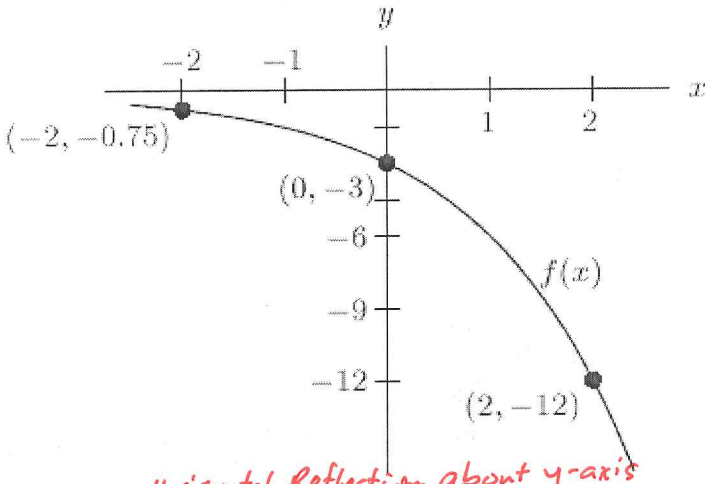
$g(f(x)) = \sqrt{1-(\cos 7x)^2}$

$= \sqrt{1-\cos^2(7x)}$

$= \sqrt{\sin^2(7x)}$

$g(f(x)) = \sin 7x$

6. Let f be defined by the following graph.



Describe the graph of $y = f(-x) + 8$. ← Horizontal Reflection about y-axis ← Vertical Translation up 8

A) A version of the graph of f flipped about the y-axis and shifted down by 8 units.
B) A version of the graph of f flipped about the y-axis and shifted up by 8 units.
C) A version of the graph of f flipped about the x-axis and shifted up by 8 units.
D) A version of the graph of f flipped about the x-axis and shifted down by 8 units.

7. Let $f(x) = \dfrac{3}{5+x}$. Use composition of functions to check/prove that $f^{-1}(x) = \dfrac{3+5x}{x}$?

$f(f^{-1}(x)) = \dfrac{3}{5+\frac{3+5x}{x}}$

$= \dfrac{3}{\frac{5x}{x}+\frac{3+5x}{x}}$

$= \dfrac{3}{\frac{10x+3}{x}}$

$= 3 \cdot \dfrac{x}{10x+3} \neq x$

NOT INVERSES
$f(f^{-1}(x)) \neq x$
$f^{-1}(f(x)) \neq x$

$f^{-1}(f(x)) = \dfrac{3+5\left(\frac{3}{5+x}\right)}{\frac{3}{5+x}}$

$= \dfrac{3+\frac{15}{5+x}}{\frac{3}{5+x}}$

$= \dfrac{\frac{3(5+x)}{5+x}+\frac{15}{5+x}}{\frac{3}{5+x}}$

$= \dfrac{\frac{15+3x+15}{5+x}}{\frac{3}{5+x}}$

$= \dfrac{3x+30}{5+x} \cdot \dfrac{5+x}{3}$

$= \dfrac{3x+30}{3}$

$= x + 10 \neq x$

8. Find $f^{-1}(240)$ if $f(x) = 80e^{2x-4}$. Round your answers to the nearest thousandth.

$240 = 80e^{2x-4}$

$3 = e^{2x-4}$

$\ln 3 = \ln(e^{2x-4})$

$\ln 3 = 2x - 4$

$\ln 3 + 4 = 2x$

$\frac{\ln 3 + 4}{2} = x$

$f^{-1}(240) = 2.549$

9. Let $f(x) = 3^x$ and $g(x) = f(f(x))$. Evaluate $g^{-1}(10)$ to 2 decimal places.

$10 = f(f(x))$

$10 = 3^{3^x}$

$\log 10 = \log(3^{3^x})$

$\log 10 = 3^x \log 3$

$\frac{\log 10}{\log 3} = 3^x$

$\log\left(\frac{\log 10}{\log 3}\right) = \log(3^x)$

$\log\left(\frac{\log 10}{\log 3}\right) = x \cdot \log 3$

$\frac{\log\left(\frac{\log 10}{\log 3}\right)}{\log 3} = x$

$.67 = x$

10. Find $f(x)$ if $f^{-1}(x) = \frac{4-3x}{2x-5}$.

$y = \frac{4-3x}{2x-5}$

$\frac{x}{1} = \frac{4-3y}{2y-5}$

$x(2y-5) = 4-3y$

$2xy - 5x = 4 - 3y$

$2xy + 3y = 5x + 4$

$y(2x+3) = 5x + 4$

$y = \frac{5x+4}{2x+3}$

$f(x) = \frac{5x+4}{2x+3}$

11. Based on the following table, could the function $f(x)$ be invertible?

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	7	9	12	14	15	16	17	18	20	22	25

YES - WOULD PASS HORIZONTAL LINE TEST → 1 TO 1 FUNCTION

12. Given $f^{-1}(x) = 300(1.03)^x$, solve $f^{-1}(x) = 350$. Round to 3 decimal places.

$350 = 300(1.03)^x$

$\frac{350}{300} = 1.03^x$

$\log\left(\frac{7}{6}\right) = \log(1.03^x)$

$\log\left(\frac{7}{6}\right) = x \cdot \log 1.03$

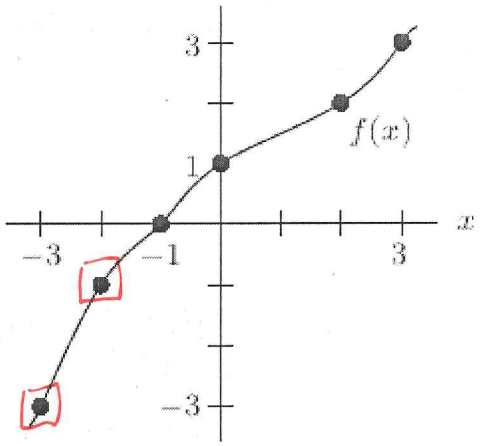
$\frac{\log\left(\frac{7}{6}\right)}{\log 1.03} = x$

or

$\frac{\log(7) - \log(6)}{\log 1.03} = x$

$x = 5.215$

13. The following figure defines a function f.



Which of the following quantities is greater?

A. $f(-3)$ = -3 (B.) $f^{-1}(-1)$ = -2

14. Suppose $f(x)=\sqrt{1-x}$. What is the range of $f^{-1}(x)$? SAME AS DOMAIN OF $f(x)$

$1-x \geq 0$

$1 \geq x$ OR $\frac{-x}{-1} \geq \frac{-1}{-1}$

$x \leq 1$

ALL REAL NUMBERS $x \leq 1$

15. Let $f(x)=x+3$ and $g(x)=x^4$. What is $3f(x)-g(x)$?

$3(x+3)-x^4$

$3x+9-x^4$

$3f(x)-g(x) = -x^4+3x+9$

16. Let $f(x)=(x^2-1)(x+2)$ and $g(x)=(x-1)(x^2-4)$. Find a simplified formula for $h(x)=\frac{3f(x)}{11g(x)}$.

↖ Difference of 2 squares

$= \frac{3(x^2-1)(x+2)}{11(x-1)(x+2)(x-2)}$

$= \frac{3(x+1)(x-1)}{11(x-1)(x-2)}$

$h(x) = \frac{3(x+1)}{11(x-2)}$